Laboratoire Kastler Brossel
Physique quantique et applications

Continuous variable quantum networks -1

Valentina Parigi

Multimode quantum optics group

Continuous Variables Quantum Complex Networks team

Okinawa School in Physics: From quantum key distribution to the quantum internet (OSP2025)

September 21, 2025 - October 3, 2025

What I do

Continuous variables

Non-linear optics

Pulsed light

Complex networks shapes

Few-photon counting

Photonic quantum computing

Continuous Variable cluster states for measurement-based protocols

**Reservoir Computing,
Variational Quantum algorithms**

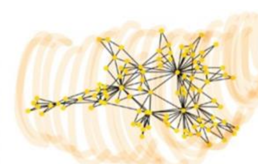
CV quantum communication networks

Multiparty quantum protocols and routing

Simulating and probing complex quantum structure

Simulating quantum environment, open quantum systems quantum thermodynamics

Probing non-Gaussian features

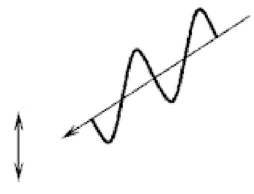


Continuous Variables Quantum Complex Networks team

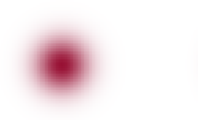
Continuous Variables

Light propagates

with different polarizations

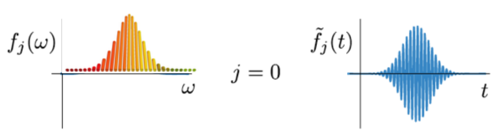


with different spatial shapes



TEM_{00}

with different spectral-temporal shapes

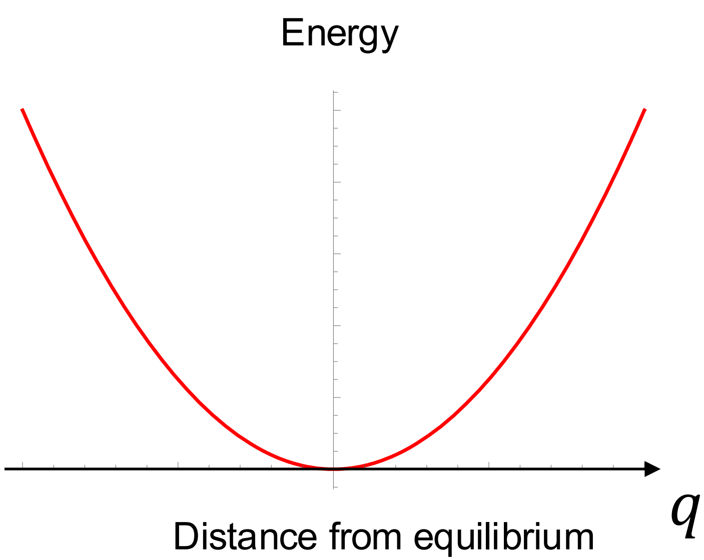


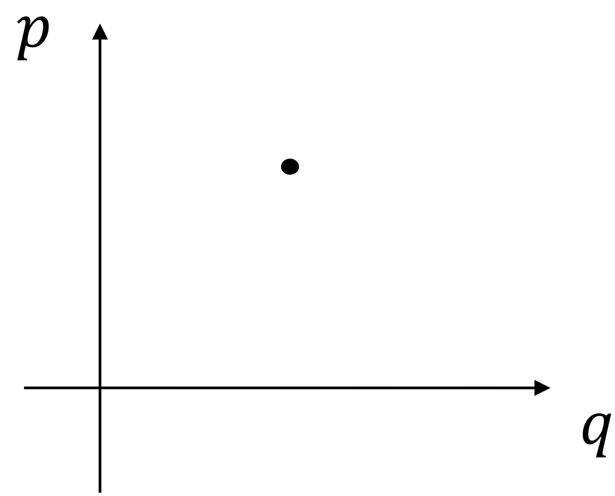
each mode of light can be described by a

quantum harmonic oscillator

$$H = \frac{p^2}{2m} + m\omega^2 \frac{q^2}{2}$$

Multimode quantum optics





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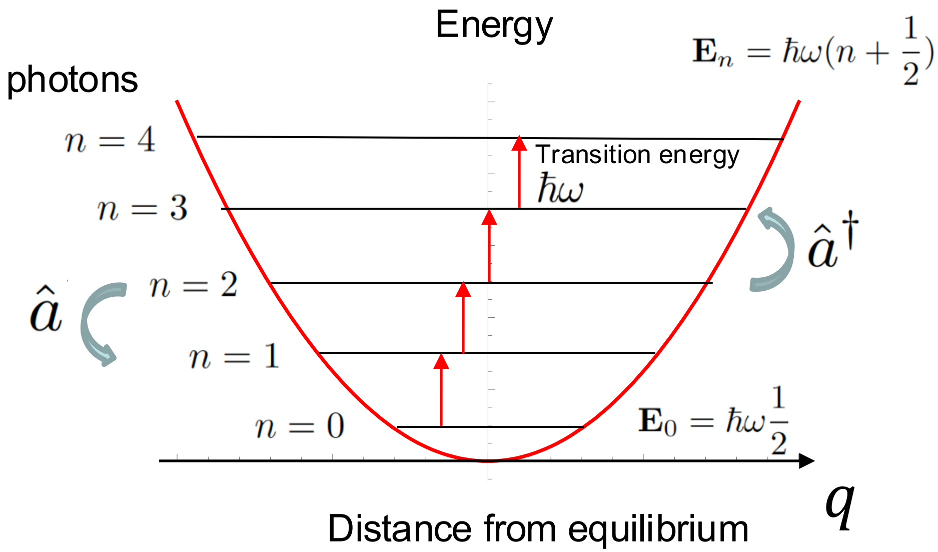
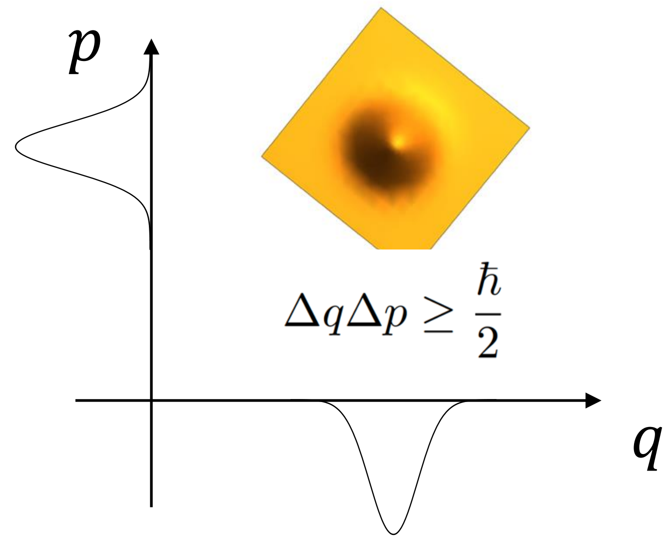




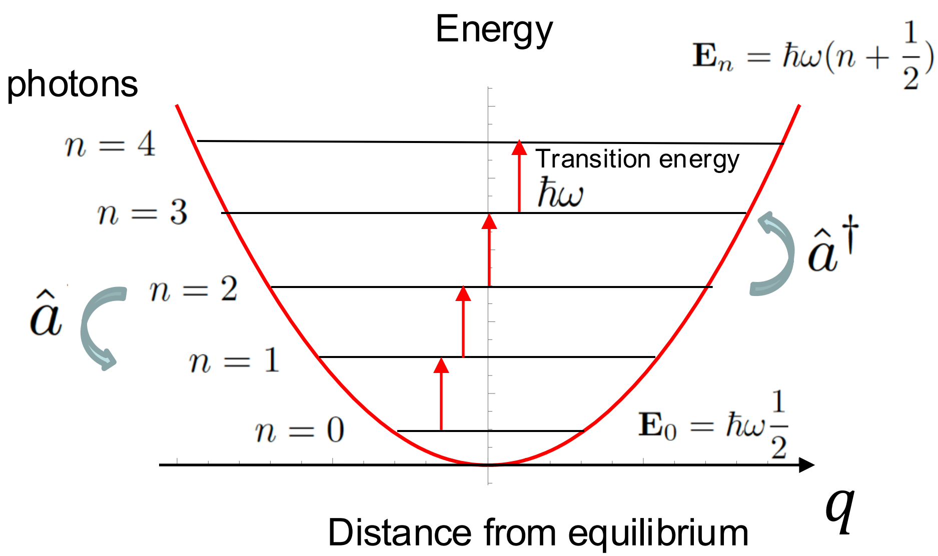
Discrete variables description ⟷ ***Continuous variables description***

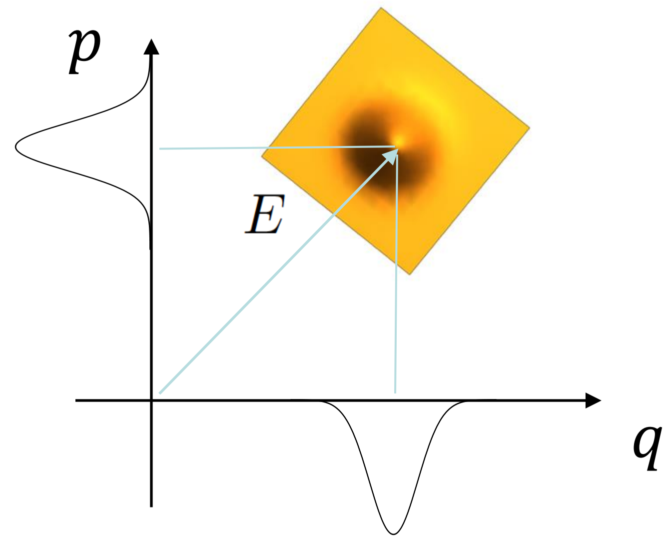
$$\hat{\rho} = \sum_{n,m} \varrho_{n,m} |n\rangle\langle m|$$

$$W(q,p)$$

⇨ each mode of light can be described by a

quantum harmonic oscillator $\hat{H} = \frac{\hat{p}^2}{2m} + m\omega^2\frac{\hat{q}^2}{2}$ $\hat{H} = \hbar\omega(\frac{1}{2} + \hat{a}^\dagger\hat{a})$





Discrete variables description

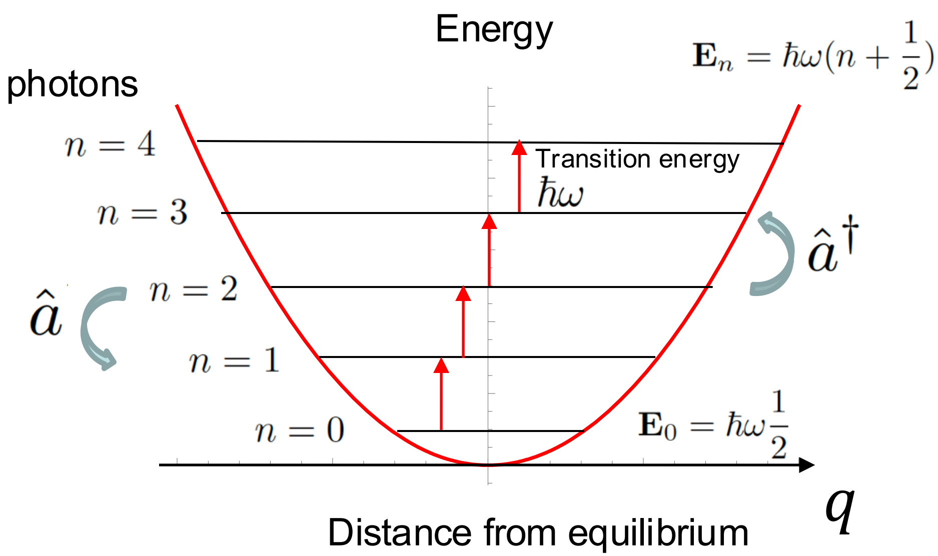
$$\hat{\rho} = \sum_{n,m} \varrho_{n,m} |n\rangle\langle m|$$

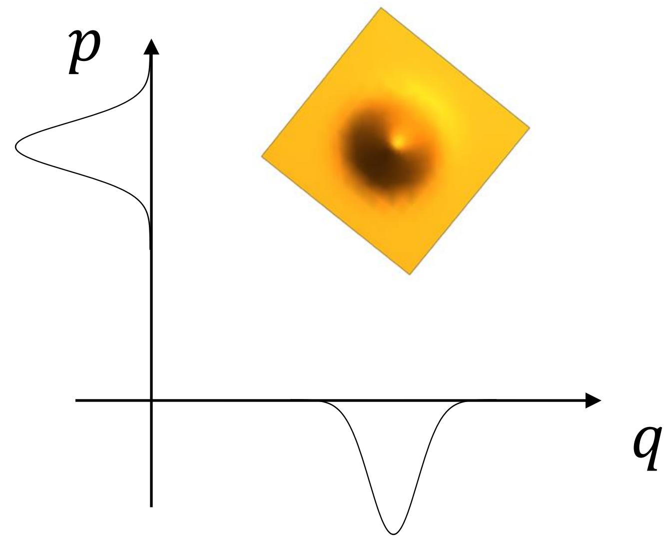
Continuous variables description

$$W(q,p)$$

each mode of light can be described by a $E = E_0 f(\vec{r},t)(\hat{q} + \imath\hat{p})$

quantum harmonic oscillator $\hat{H} = \frac{\hbar\omega}{2}(\hat{p}^2 + \hat{q}^2)$ $\hat{H} = \hbar\omega(\frac{1}{2} + \hat{a}^\dagger\hat{a})$





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$$\hat{\rho} = \sum_{n,m} \varrho_{n,m} |n\rangle\langle m|$$

Continuous variables description

$$W(q,p)$$

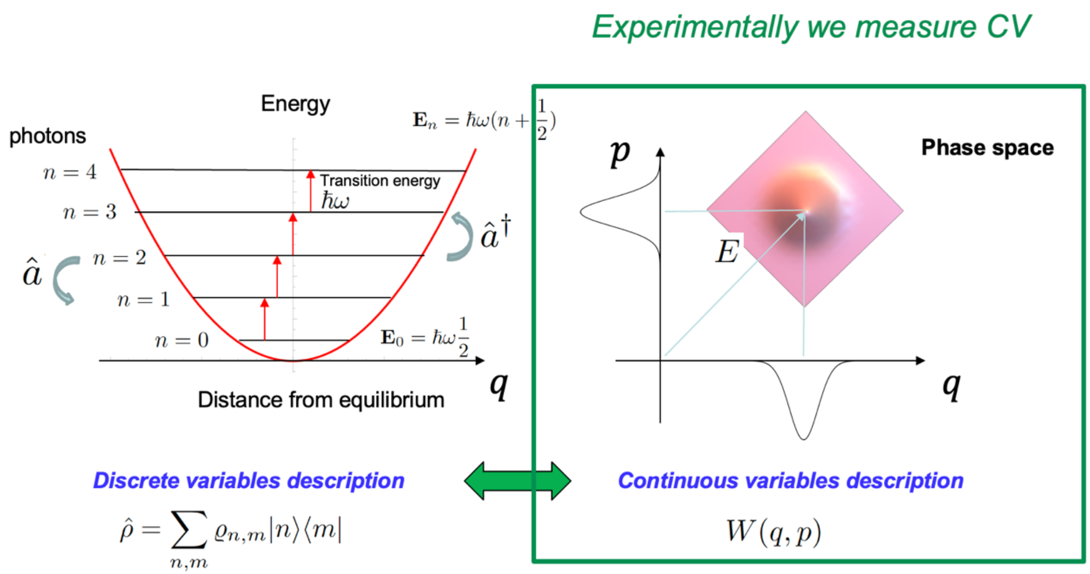
$$W(q,p) = \frac{1}{2\pi\sigma_0^2} \int e^{iyp/\sigma_0^2} \langle q-y|\hat{\rho}|q+y\rangle \, dy$$

$\sigma_0^2 = 1$

Wigner function

Continuous Variables

Continuous variables



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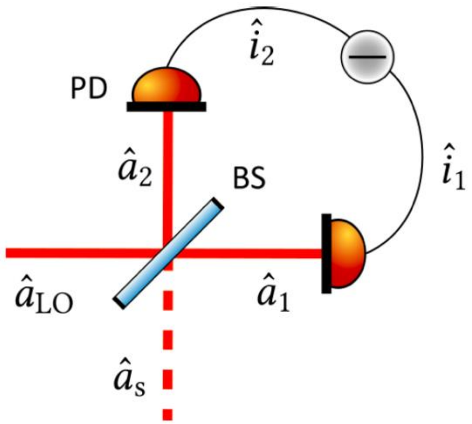
quantum harmonic oscillator $\hat{H} = \frac{\hbar\omega}{2}(\hat{p}^2 + \hat{q}^2)$ $\hat{H} = \hbar\omega(\frac{1}{2} + \hat{a}^\dagger\hat{a})$

- Room temperature coherent detection (mainly homodyne)

Homodyne detection



It consists in interfering on a balanced beamsplitter the signal field with a bright field called the local oscillator (LO)

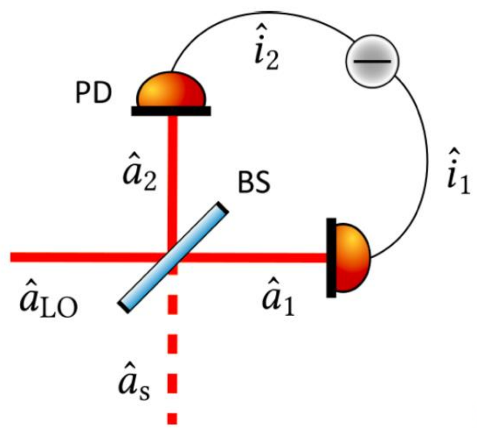
We consider that the time-frequency modes of the input fields are matched

$$\begin{cases} \hat{a}_1 = \frac{1}{\sqrt{2}}(\hat{a}_{\mathrm{LO}} + \hat{a}_{\mathrm{s}}) \\ \hat{a}_2 = \frac{1}{\sqrt{2}}(\hat{a}_{\mathrm{LO}} - \hat{a}_{\mathrm{s}}) \end{cases}$$

the currents of the two photodiode are proportional to the light intensity, so to the photon number

$$\begin{cases} \hat{i}_1 \propto \hat{n}_1 = \frac{1}{2}(\hat{a}_{\mathrm{LO}}^\dagger \hat{a}_{\mathrm{LO}} + \hat{a}_{\mathrm{LO}}^\dagger \hat{a}_{\mathrm{s}} + \hat{a}_{\mathrm{s}}^\dagger \hat{a}_{\mathrm{LO}} + \hat{a}_{\mathrm{s}}^\dagger \hat{a}_{\mathrm{s}}) \\ \hat{i}_2 \propto \hat{n}_2 = \frac{1}{2}(\hat{a}_{\mathrm{LO}}^\dagger \hat{a}_{\mathrm{LO}} - \hat{a}_{\mathrm{LO}}^\dagger \hat{a}_{\mathrm{s}} - \hat{a}_{\mathrm{s}}^\dagger \hat{a}_{\mathrm{LO}} + \hat{a}_{\mathrm{s}}^\dagger \hat{a}_{\mathrm{s}}) \end{cases}$$

Homodyne detection



It consists in interfering on a balanced beamsplitter the signal field with a bright field called the local oscillator (LO)

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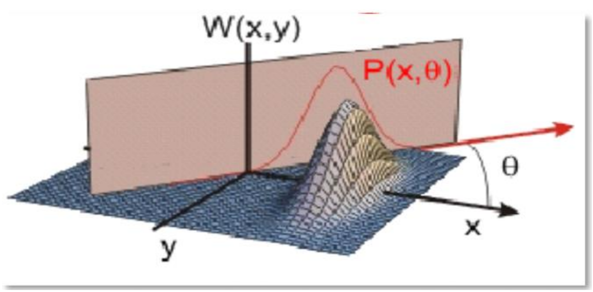
$$\hat{i}_d = \hat{i}_1 - \hat{i}_2$$

$$\hat{i}_d \propto \hat{a}^{\dagger}_{LO}\hat{a}_s + \hat{a}^{\dagger}_s\hat{a}_{LO}$$

$$\langle \hat{n}_{LO} \rangle \gg \langle \hat{n}_s \rangle \qquad \hat{a}_{LO} = \alpha_{LO} + \cancel{\delta\hat{a}_{LO}} \qquad \alpha_{LO} = |\alpha_{LO}|e^{i\theta}$$

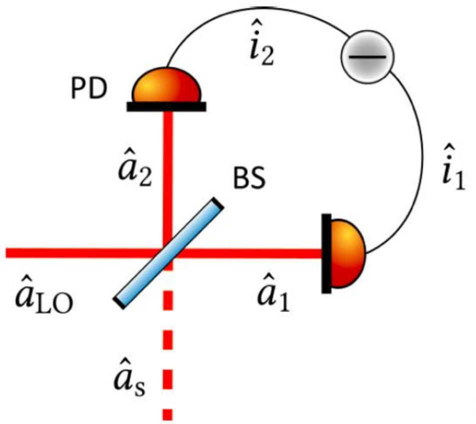
$$\hat{i}_d \propto |\alpha_{LO}|\, \hat{q}^{\theta}_s$$

with $\quad \hat{q}^{\theta}_s = \sigma_0 \hat{a}^{\dagger} e^{i\theta} + \sigma_0 \hat{a} e^{-i\theta}$



homodyne detection allows us to directly measure quadratures so their statistics

Homodyne detection



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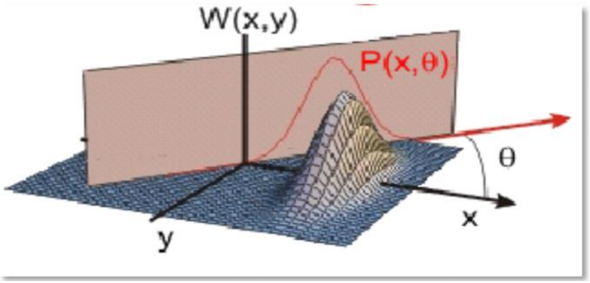
$$\hat{i}_d = \hat{i}_1 - \hat{i}_2$$

$$\hat{i}_d \propto \hat{a}_{LO}^\dagger \hat{a}_s + \hat{a}_s^\dagger \hat{a}_{LO}$$

probability of measuring outcome q_s^θ *is* $\langle q_s^\theta | \hat{\rho}_s | q_s^\theta \rangle$

$$\hat{q}_s^\theta = \sigma_0 \hat{a}^\dagger e^{i\theta} + \sigma_0 \hat{a} e^{-i\theta}$$

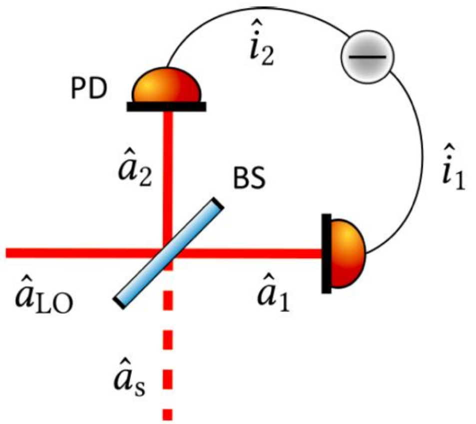
POVM $\hat{\Pi}_{HD}(q_s^\theta) = |q_s^\theta\rangle\langle q_s^\theta|$



homodyne detection allows us to directly measure quadratures so their statistics

Measuring Continuous Variables

From ***Homodyne detection*** ⇨ ***Wigner Function***



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From the measure

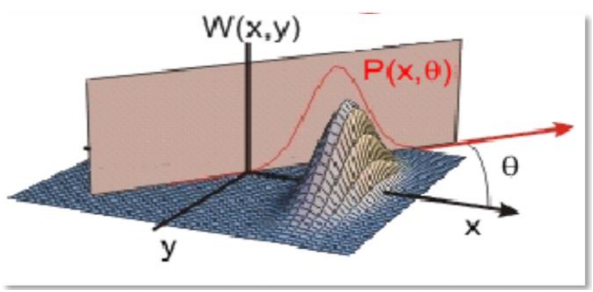
$$\{P(x_\theta)\}_{\theta=0\rightarrow 2\pi}$$

$$P(x_\theta) = \int W(x_\theta, y_\theta) dy$$

It is possible to reconstruct

$$W(x,y)$$

Inverse Radon transform



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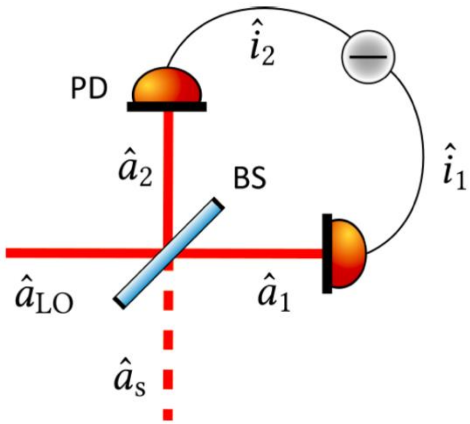

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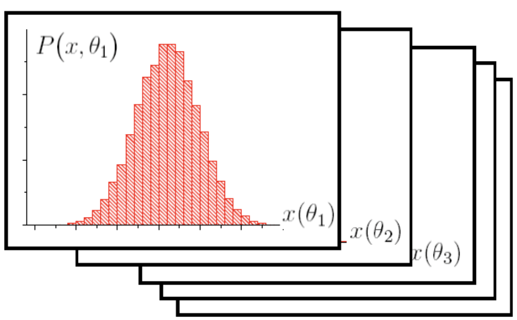
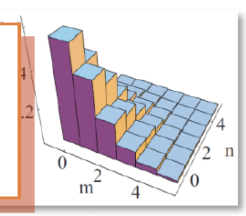


Complete set of quadrature distributions

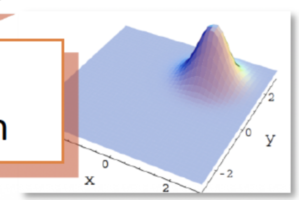
Maximum likelihood estimation

Tomographic reconstruction

Density matrix elements ρ_{nm}



Wigner function



A.I. Lvovsky, *J. Opt. B: Quantum Semiclassical Opt.* **6**, 556 (2004)

Z. Hradil, D. Mogilevtsev, J. Rehacek, *PRL* **96**, 230401 (2006)

$[\hat{a},\hat{a}^+] = 1$ always

If

$$\begin{cases} \hat{q} = \hat{a}^+ + \hat{a} \\ \hat{p} = i(\hat{a}^+ - \hat{a}) \end{cases}$$

$\Downarrow$

$$[\hat{q},\hat{p}] = 2i$$

$$\begin{cases} \hat{a}^+ = \frac{1}{2}(\hat{q} - i\hat{p}) \\ \hat{a} = \frac{1}{2}(\hat{q} + i\hat{p}) \end{cases}$$

$\langle 0|\hat{q}^2|0\rangle = 1$

$\langle 0|\hat{p}^2|0\rangle = 1$

$\langle 0|$ ↓ vacuum state

If

$$\begin{cases} \hat{q} = \frac{1}{\sqrt{2}}(\hat{a}^+ + \hat{a}) \\ \hat{p} = \frac{i}{\sqrt{2}}(\hat{a}^+ - \hat{a}) \end{cases}$$

$\Downarrow$

$$[\hat{q},\hat{p}] = i$$

$$\begin{cases} \hat{a}^+ = \frac{1}{\sqrt{2}}(\hat{q} - i\hat{p}) \\ \hat{a} = \frac{1}{\sqrt{2}}(\hat{q} + i\hat{p}) \end{cases}$$

$\langle 0|\hat{q}^2|0\rangle = 1/2$

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If

$$\begin{cases} \hat{q} = \frac{1}{2}(\hat{a}^+ + \hat{a}) \\ \hat{p} = \frac{i}{2}(\hat{a}^+ - \hat{a}) \end{cases}$$

$\Downarrow$

$$[\hat{q},\hat{p}] = i/2$$

$$\begin{cases} \hat{a}^+ = (\hat{q} - i\hat{p}) \\ \hat{a} = (\hat{q} + i\hat{p}) \end{cases}$$

$\langle 0|\hat{q}^2|0\rangle = 1/4$

$\langle 0|\hat{p}^2|0\rangle = 1/4$

$\langle 0|$ ↓ vacuum state

Generalized formalism

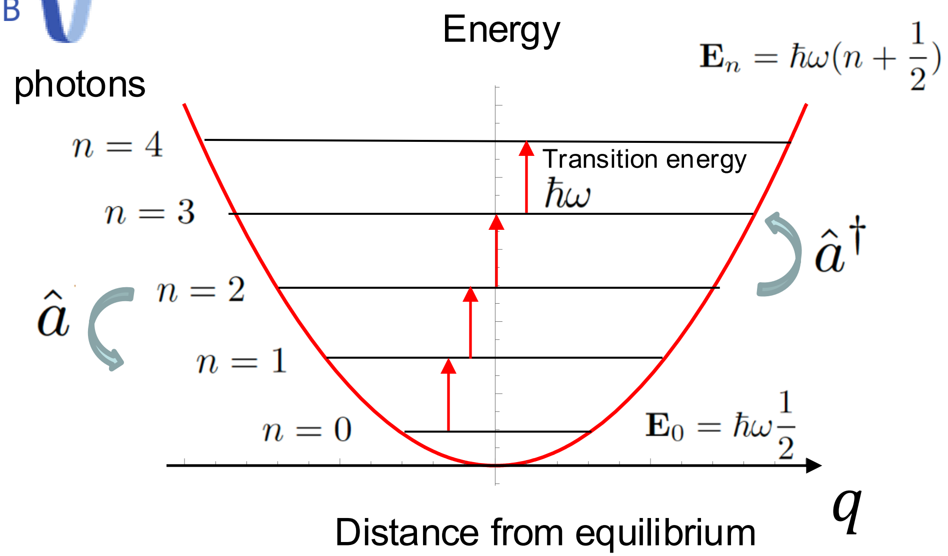
$$\langle 0|\hat{q}^2|0\rangle = \sigma_0^2$$
$$\langle 0|\hat{p}^2|0\rangle = \sigma_0^2$$

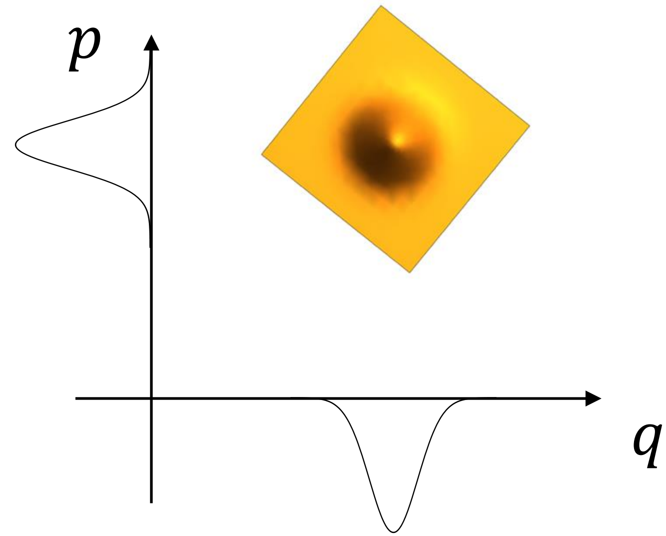
$$[\hat{q},\hat{p}] = 2i\sigma_0^2$$

$$\begin{cases} \hat{q} = \sigma_0(\hat{a}+\hat{a}^\dagger) \\ \hat{p} = i\sigma_0(\hat{a}^\dagger-\hat{a}) \end{cases} \qquad \begin{cases} \hat{a} = \dfrac{\hat{q}+i\hat{p}}{2\sigma_0} \\ \hat{a} = \dfrac{\hat{q}-i\hat{p}}{2\sigma_0} \end{cases}$$

$$W(q,p) = \frac{1}{2\pi\sigma_0^2}\int e^{iyp/\sigma_0^2}\langle q-y|\hat{\rho}|q+y\rangle\, dy$$

Multimode quantum optics





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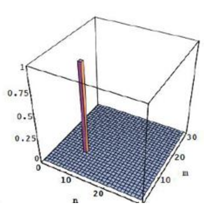
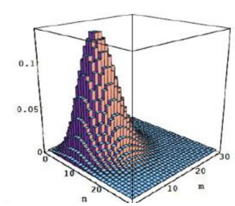
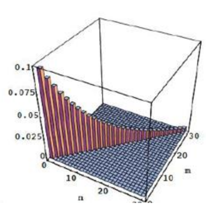
$$W(q, p)$$



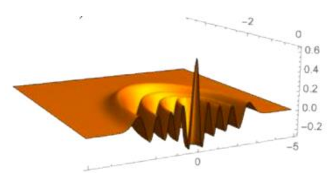
Fock



Coherent



Thermal



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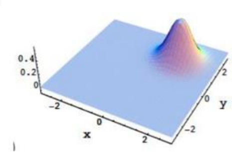

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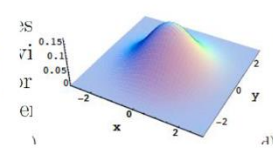

Thermal

Fully equivalent (sometimes one is more practical than the other..)

Continuous Variables Quantum Information

Quantum information

Discrete variables encoding

0 1 0 0 0 1 1 1 0 1

Continuous variables encoding

$\{x_i\} \in \mathcal{R}$

superposition

$| \psi \rangle = \alpha|0\rangle + \beta|1\rangle$

$| \psi \rangle = \int f(x)|x\rangle dx$

entanglement: quantum correlations

$| \psi \rangle = \frac{1}{\sqrt{2}}(|0\rangle_S|1\rangle_I + |1\rangle_S|0\rangle_I)$

$|\psi\rangle \sim \delta(q_s - q_i)\delta(p_s + p_i)$

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"any" operation

universal gate set

$\{H, R(\pi/2), CNOT, R(\pi/4)\}$

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$$U(t) = \exp\left(-\frac{it}{\hbar}H(\mathbf{q},\mathbf{p})\right)$$

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"any" operation -> any unitary (Hamiltonian) evolution

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interlude

How it works : if you can implement H_1 and H_2, you can implement the commutator

Baker-Campbell-Hausdorff formulae

$$e^{itH_2}e^{itH_1}e^{-itH_2}e^{-itH_1} = e^{-t^2[H_2,H_1]} + O\left(t^3\right)$$

S. Lloyd and S. L. Braunstein Phys. Rev. Lett., 82 (1999)

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Quantum information

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"any" operation -> any unitary (Hamiltonian) evolution

universal unitary (Hamiltonian) set

$\{e^{\imath \hat{q} s}, e^{\imath \hat{q}^2 s}, e^{\imath \frac{\pi}{4}(\hat{q}^2+\hat{p}^2)}\}$

single-mode Gaussian evolution

Quantum information

Discrete variables encoding	*Continuous variables encoding*
0 1 0 0 0 1 1 1 0 1	$\{x_i\} \in \mathcal{R}$

superposition

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"any" operation $\qquad$ *"any" operation -> any unitary (Hamiltonian) evolution*

universal gate set $\qquad$ *universal unitary (Hamiltonian) set*

$\{H, R(\pi/2), CNOT, R(\pi/4)\}$ $\qquad$ $\{e^{\imath\hat{q}s}, e^{\imath\hat{q}^2 s}, e^{\imath\frac{\pi}{4}(\hat{q}^2+\hat{p}^2)}, \quad e^{\imath\hat{q}^3 s}\}$

cubic gate
non – Gaussian

Quantum information

Discrete variables encoding

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universal unitary (Hamiltonian) set

$\{H, R(\pi/2), CNOT, R(\pi/4)\}$

$\{e^{\imath \hat{q} s}, e^{\imath \hat{q}^2 s}, e^{\imath \frac{\pi}{4}(\hat{q}^2+\hat{p}^2)}, e^{\imath \hat{q}_1 \otimes \hat{q}_2}, e^{\imath \hat{q}^3 s}\}$

two mode entangling gate Cz

Quantum information

Discrete variables encoding	*Continuous variables encoding*
0 1 0 0 0 1 1 1 0 1	$\{x_i\} \in \mathcal{R}$

superposition

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$|\psi\rangle = \int f(x)|x\rangle dx$$

entanglement: quantum correlations

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle_S|1\rangle_I + |1\rangle_S|0\rangle_I)$$

$$|\psi\rangle \sim \delta(q_s - q_i)\delta(p_s + p_i)$$

"any" operation

universal gate set

$$\{H, R(\pi/2), CNOT, R(\pi/4)\}$$

2-photon gate

"any" operation -> any unitary (Hamiltonian) evolution

universal unitary (Hamiltonian) set

$$\{e^{\imath\hat{q}s}, e^{\imath\hat{q}^2 s}, e^{\imath\frac{\pi}{4}(\hat{q}^2+\hat{p}^2)}, e^{\imath\hat{q}_1\otimes\hat{q}_2}, e^{\imath\hat{q}^3 s}\}$$

cubic gate
non – Gaussian

Hard to do

Implementation in optics

Quantum information

Discrete variables encoding		*Continuous variables encoding*
0 1 0 0 0 1 1 1 0 1		$\{x_i\} \in \mathcal{R}$
$\vert \psi \rangle = \alpha\vert 0\rangle + \beta\vert 1\rangle$	*superposition*	$\vert \psi \rangle = \int f(x)\vert x\rangle dx$
$\vert \psi \rangle = \frac{1}{\sqrt{2}}(\vert 0\rangle_S\vert 1\rangle_I + \vert 1\rangle_S\vert 0\rangle_I)$	*entanglement: quantum correlations*	$\vert \psi\rangle \sim \delta(q_s - q_i)\delta(p_s + p_i)$

Real advantage over classical

Need of many photons	*Need of many quantum modes*
Probabilistic but higher fidelity	*Deterministic but lower fidelity*
$\{H, R(\pi/2), CNOT, R(\pi/4)\}$	$\{e^{\imath \hat{q} s}, e^{\imath \hat{q}^2 s}, e^{\imath \frac{\pi}{4}(\hat{q}^2+\hat{p}^2)}, e^{\imath \hat{q}_1 \otimes \hat{q}_2}, e^{\imath \hat{q}^3 s}\}$
2-photon gate	*cubic gate* *non – Gaussian*

Hard to do

Implementation in optics

And not speaking about error correction

Quantum information

Discrete variables encoding

0 1 0 0 0 1 1 1 0 1

Continuous variables encoding

$\{x_i\} \in \mathcal{R}$

superposition

$| \psi \rangle = \alpha|0\rangle + \beta|1\rangle$

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$$|\psi\rangle \sim \delta(q_s - q_i)\delta(p_s + p_i)$$

circuit model

$\{H, R(\pi/2), CNOT, R(\pi/4)\}$

2-photon gate

$\{e^{\imath \hat{q} s}, e^{\imath \hat{q}^2 s}, e^{\imath \frac{\pi}{4}(\hat{q}^2+\hat{p}^2)}, e^{\imath \hat{q}_1 \otimes \hat{q}_2}, e^{\imath \hat{q}^3 s}\}$

cubic gate
non – Gaussian

Hard to do

Implementation in optics

And not speaking about error correction

Multimode quantum optics

Good platform for

Quantum information

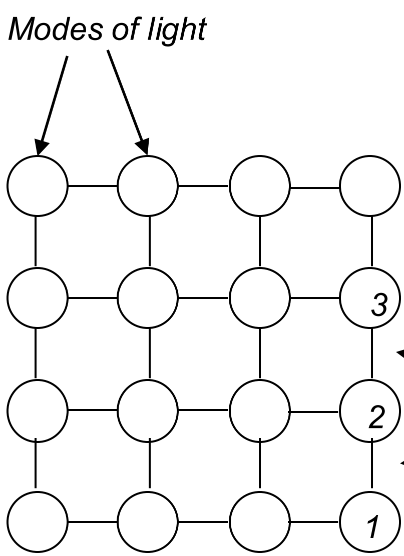
Continuous variables encoding

$$\{x_i\} \in \mathcal{R}$$

superposition $| \psi \rangle = \int f(x) |x\rangle dx$

entanglement: quantum correlations

$$|\psi\rangle \sim \delta(q_s - q_i)\delta(p_s + p_i)$$



measurement based (one way) model

concept

⇒ *Large entangled state: cluster state*

CV entanglement correlations

$$\delta(p_2 - q_1 - q_3)$$

Good platform for

Quantum information

Continuous variables encoding

$$\{x_i\} \in \mathcal{R}$$

superposition

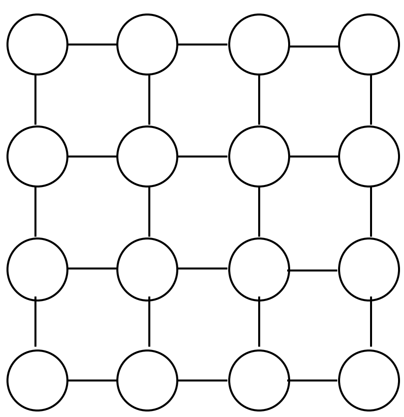
$$|\,\psi\,\rangle = \int f(x)|x\rangle dx$$

entanglement: quantum correlations

$$|\psi\rangle \sim \delta(q_s - q_i)\delta(p_s + p_i)$$

measurement based (one way) model

concept



Multimode quantum optics

Good platform for

Quantum information

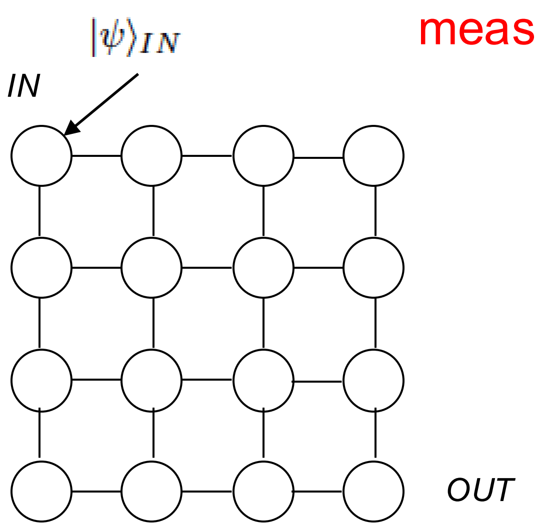
Continuous variables encoding

$$\{x_i\} \in \mathcal{R}$$

superposition $|\psi\rangle = \int f(x)|x\rangle dx$

entanglement: quantum correlations

$$|\psi\rangle \sim \delta(q_s - q_i)\delta(p_s + p_i)$$



measurement based (one way) model

concept

Good platform for

Quantum information

Continuous variables encoding

$$\{x_i\} \in \mathcal{R}$$

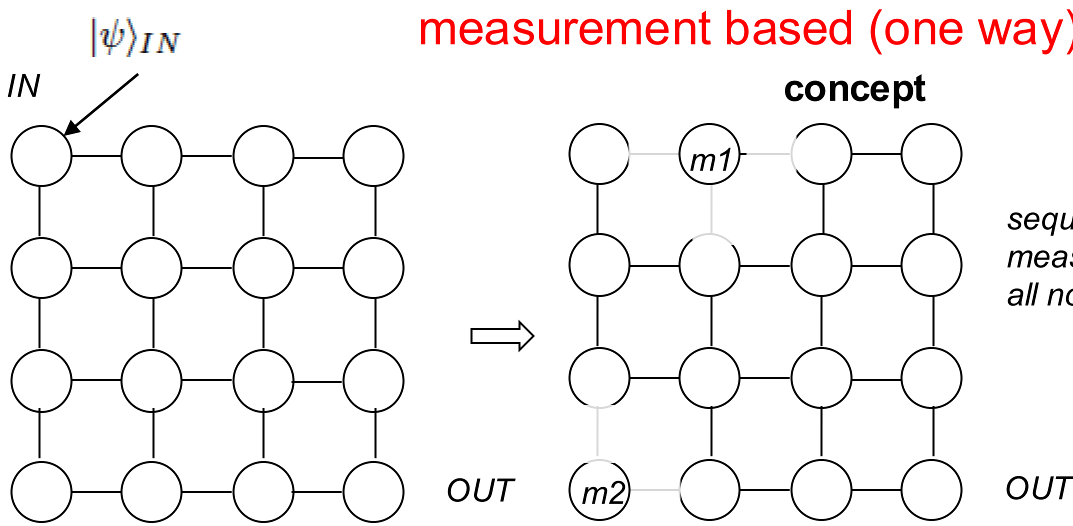
superposition

$$|\,\psi\,\rangle = \int f(x)|x\rangle dx$$

entanglement: quantum correlations

$$|\psi\rangle \sim \delta(q_s - q_i)\delta(p_s + p_i)$$

measurement based (one way) model



sequential measurement at all nodes

Use measurement results to:
- decide next measurement
- correct for the desired operation

Multimode quantum optics

Good platform for

Quantum information

Continuous variables encoding

$$\{x_i\} \in \mathcal{R}$$

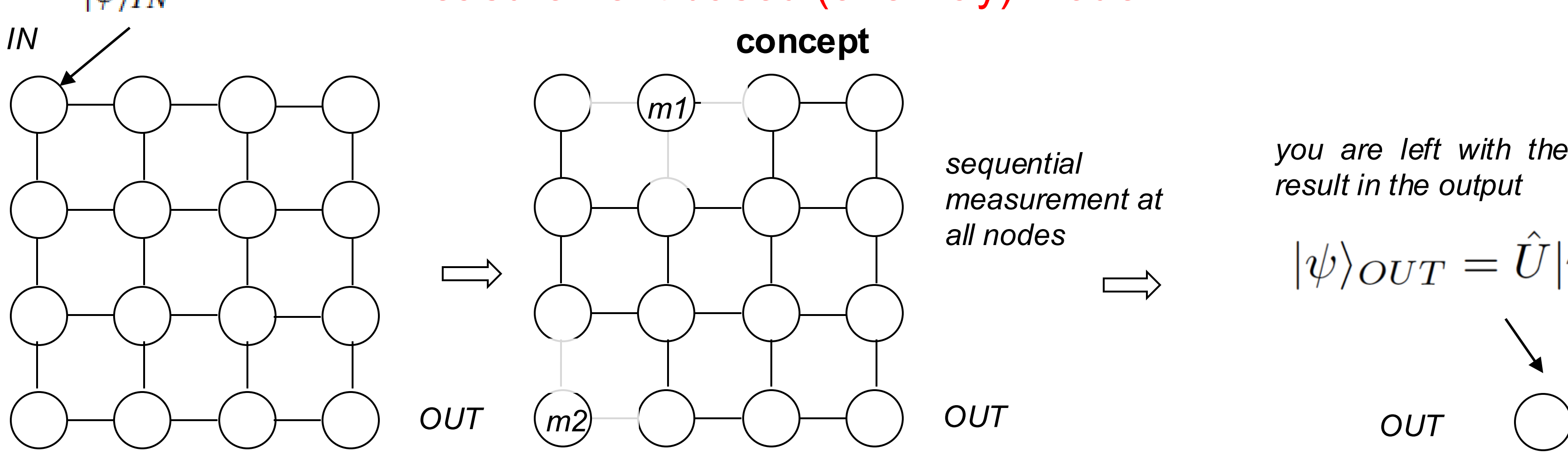
superposition

$$|\psi\rangle = \int f(x)|x\rangle dx$$

entanglement: quantum correlations

$$|\psi\rangle \sim \delta(q_s - q_i)\delta(p_s + p_i)$$

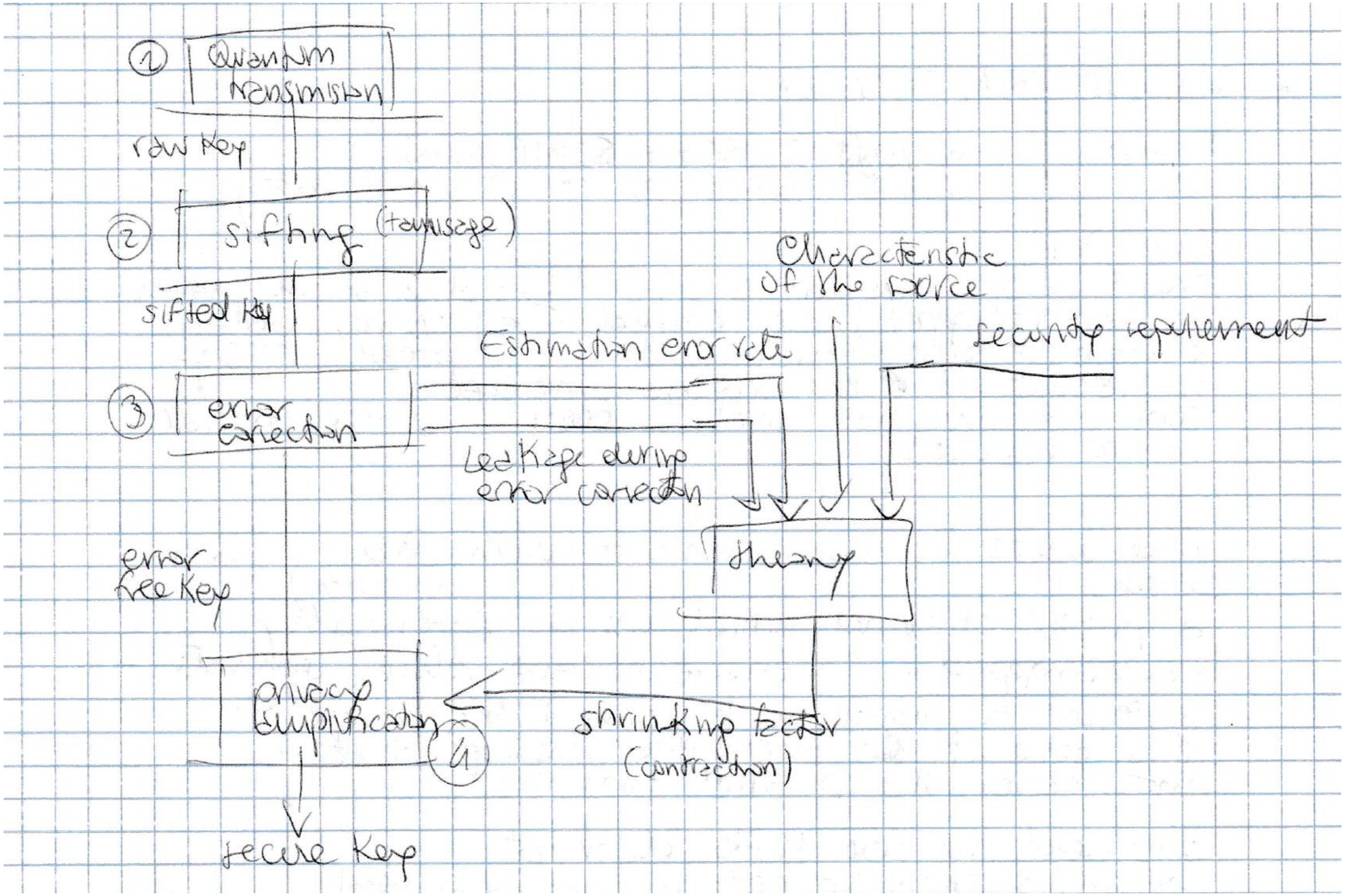
measurement based (one way) model



Continuous Variable
Basic Protocols
(before looking at the networks)

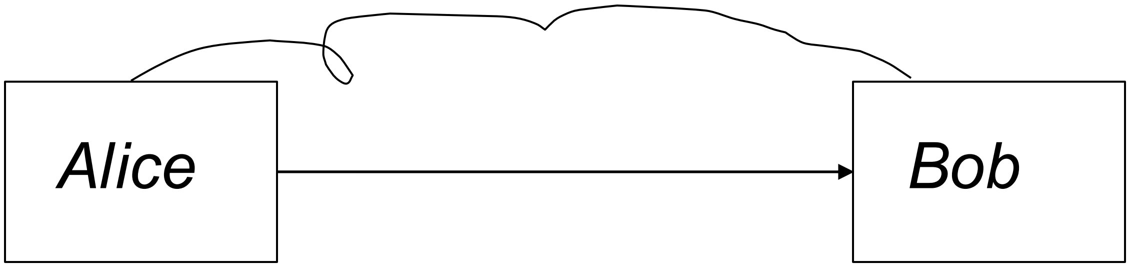
Continuous Variable
Basic Protocols 1
CV-QKD

Quantum key distribution scheme



What it should look like if you are an expert (I am not!)

CV Quantum key distribution idea



1) Alice draws two random numbers x_A *and* p_A *from a Gaussian distribution of mean zero and variance* $V_A N_0$

$N_0 = \sigma_0^2$

2) She sends the coherent state $|x_A + i p_A\rangle$ *to Bob*

3) Bob randomly chooses to measure either the quadrature x or p

4) Later Bob informs Alice by using a public channel, so that they can discard the irrelevant date

Why it works -> the two quadrature (q & p , or x & p) do not commute -> you don't need too much "quantum" states

CV Quantum key distribution idea

letters to nature

Quantum key distribution using gaussian-modulated coherent states

Frédéric Grosshans*, Gilles Van Assche†, Jérôme Wenger*, Rosa Brouri*, Nicolas J. Cerf† & Philippe Grangier*

NATURE | VOL 421 | 16 JANUARY 2003 | www.nature.com/nature

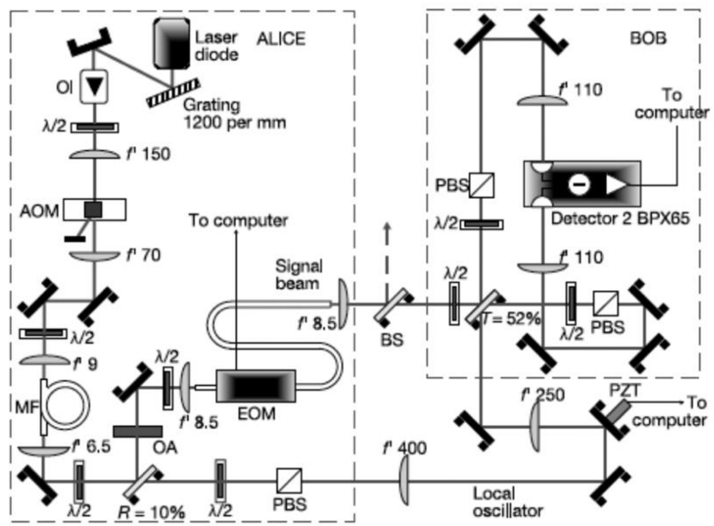


Figure 1 Experimental set-up. Laser diode, SDL 5412 (780 nm); OI, optical isolator; λ/2, half-wave plate; AOM, acousto-optic modulator; MF, polarization maintaining single-mode fibre; OA, optical attenuator; EOM, electro-optic amplitude modulator; PBS, polarizer; BS, beam splitter; PZT, piezoelectric transducer. Focal lengths (*f*′) are given in millimetres. *R* and *T* are reflection and transmission coefficients.

CV Quantum key distribution idea

LKB

letters to nature

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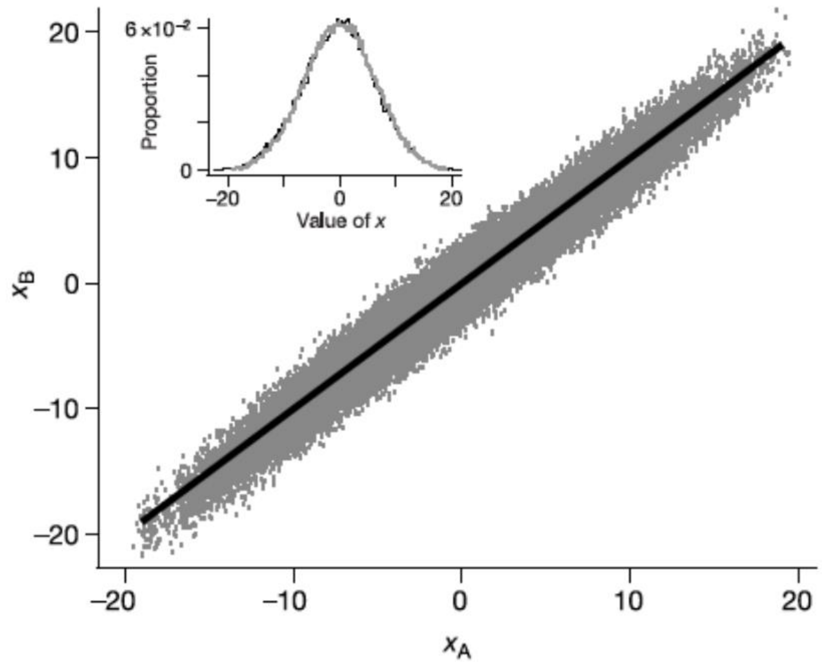


CV Quantum key distribution idea

P. Grangier, GDR-IQFA Colloquium 2013

Coherent state continuous variables QKD protocol

- Key information encoded in both quadratures of a coherent state

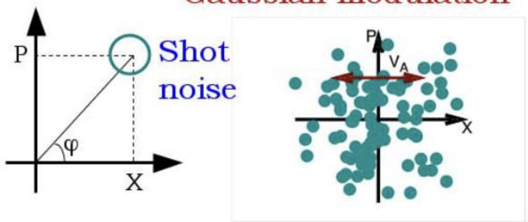
Eve

Alice

Quantum channel (T, ε)
+ Classical Channel (auth.)

Bob

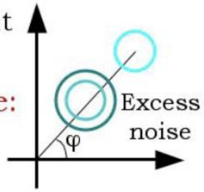
Gaussian modulation



Random measurement of the quadrature of each coherent state (with efficiency η)

Total channel-added noise:

$$\chi = \underbrace{1/T - 1}_{\text{equivalent to photon loss}} + \underbrace{\varepsilon}_{\text{equivalent to errors}}$$



- Bob reveals measurement choice
- Alice and Bob share a set of Gaussian correlated data
- Further communication to calculate channel parameters and derive secret key based on Bob's data → **reverse reconciliation**

F. Grosshans et al, Phys. Rev. Lett. 88, 057902 (2002) & Nature 421, 238 (2003)

CV Quantum key distribution idea

P. Grangier, GDR-IQFA Colloquium 2013

QKD protocol using coherent states with gaussian amplitude and phase modulation

Efficient transmission of information using continuous variables ?

-> Shannon's formula (1948) : the mutual information I_{AB} (unit : bit / symbol) for a gaussian channel with additive noise is given by

$$\boxed{I_{AB} = 1/2 \ \log_2 [\, 1 + V(\text{signal}) / V(\text{noise}) \,]}$$

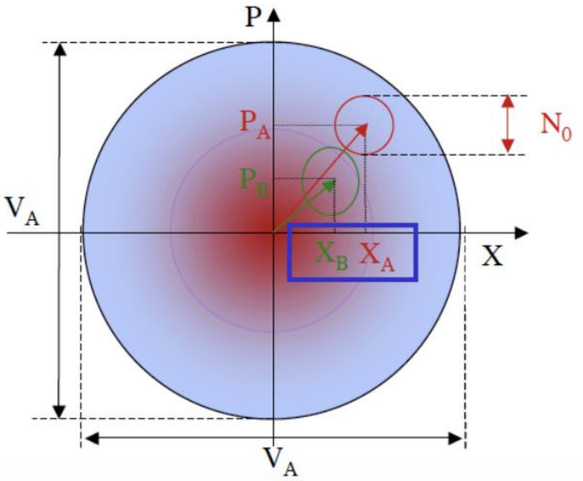
Reminder : I(X; Y) =
H(X) - H(X | Y) =
H(Y) - H(Y| X) =
H(X) + H(Y) - H(X; Y)

(a) Alice chooses X_A and P_A within two random gaussian distributions.

(b) Alice sends to Bob the coherent state $| X_A + i\, P_A >$

(c) Bob measures either X_B or P_B

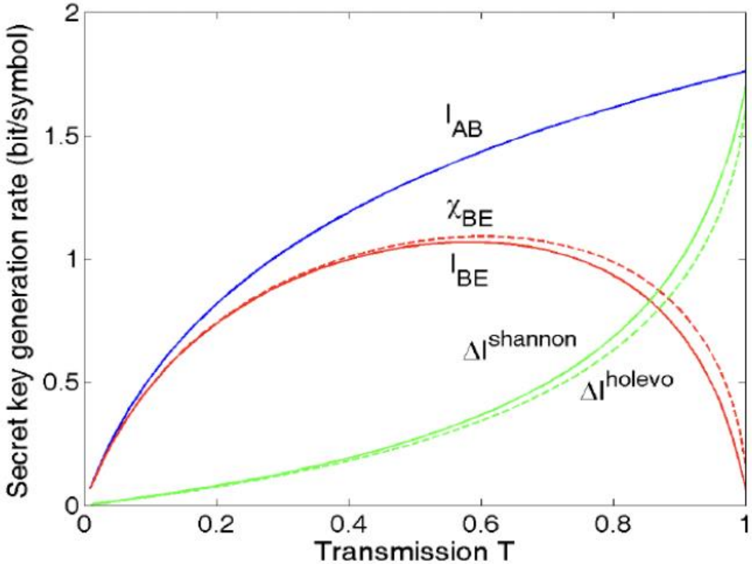
(d) Bob and Alice agree on the basis choice (X or P), and keep the relevant values.



CV Quantum key distribution idea

P. Grangier, GDR-IQFA Colloquium 2013

Security of coherent state CV-QKD : collective attacks



Alice-Bob mutual information : I_{AB}

Eve-Bob mutual information :

I_{BE} (Shannon : individual attacks)

χ_{BE} (Holevo : collective attacks)

Bounded from channel evaluation !

Secret Key Rate :

$\Delta I = I_{AB} - I_{BE}$ (Shannon)

$\Delta I = I_{AB} - \chi_{BE}$ (Holevo)

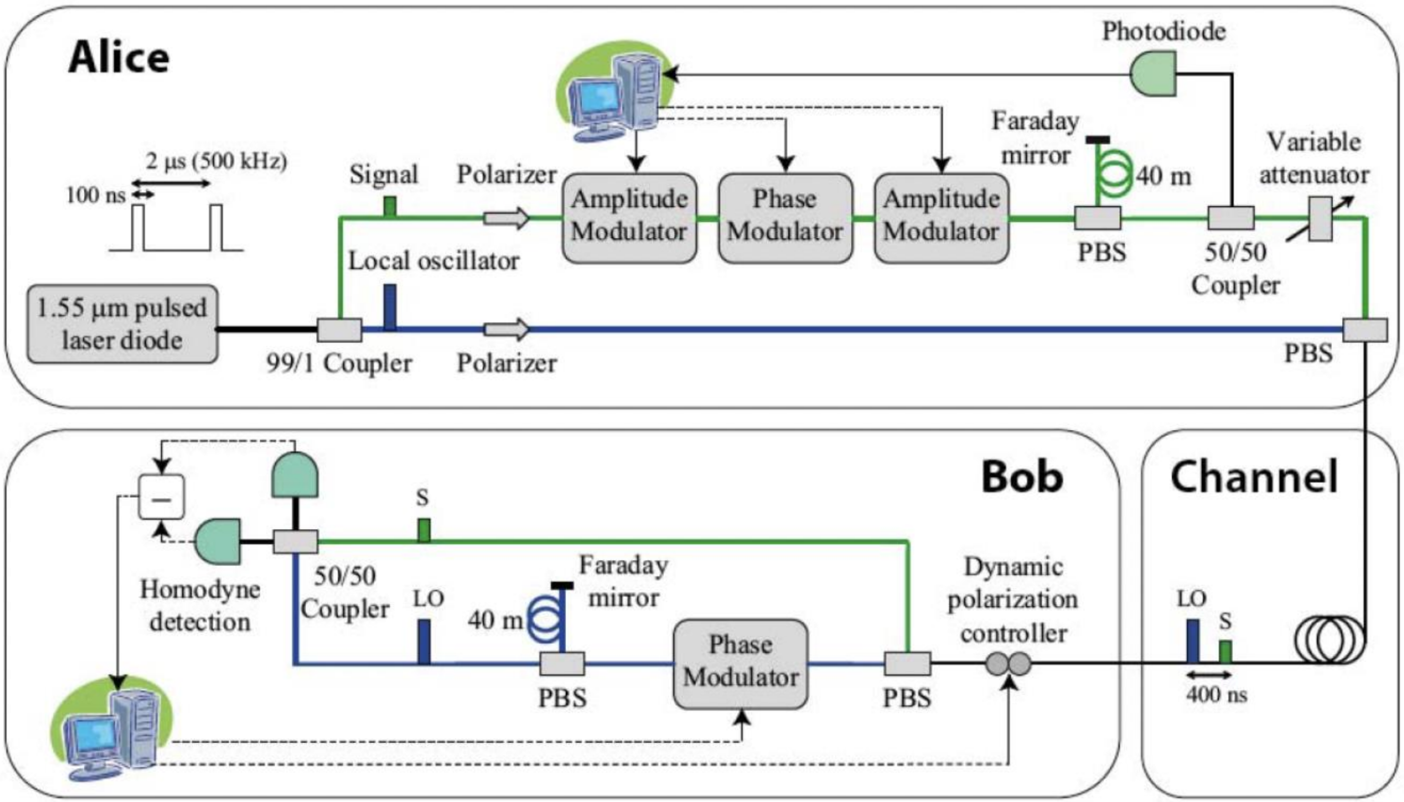
- For both individual and collective attacks Gaussian attacks are optimal → Alice and Bob consider Eve's attacks Gaussian and estimate her information using the Shannon quantity I_{BE} or the Holevo quantity χ_{BE}

Fig : $V_A = 21$ (shot noise units)
$\varepsilon = 0.005$ (shot noise units), $\eta = 0.5$

M. Navasqués et al, Phys. Rev. Lett. 97, 190502 (2006)
R. García-Patrón et al, Phys. Rev. Lett. 97, 190503 (2006)

CV Quantum key distribution idea

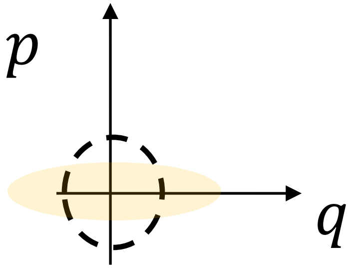
All-fibered CVQKD @ 1550 nm



Field test of a continuous-variable quantum key distribution prototype
S Fossier, E Diamanti, T Debuisschert, A Villing, R Tualle-Brouri and P Grangier
New J. Phys. 11 No 4, 04502 (April 2009)

Continuous Variable
Basic Protocols 2
CV-quantum teleportation

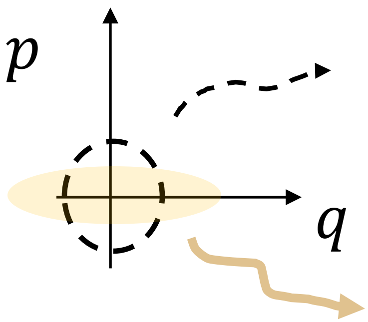
CV Quantum teleportation. The resource ?



squeezed state of light

LKB

CV Quantum teleportation. The resource ?



vacuum

$\Delta q \, \Delta p = 1$ $\qquad$ $\Delta q = \Delta p = 1$

squeezed state of light

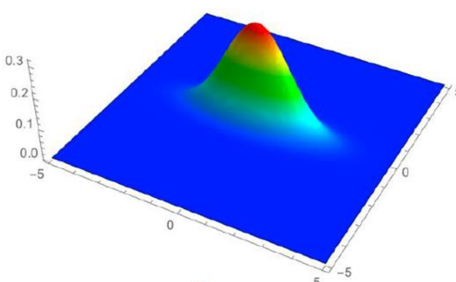
$\Delta q \, \Delta p = 1$ $\qquad$ $\Delta q < 1 \quad \Delta p > 1$

$$|\zeta\rangle = \hat{S}(\zeta)\,|0\rangle$$

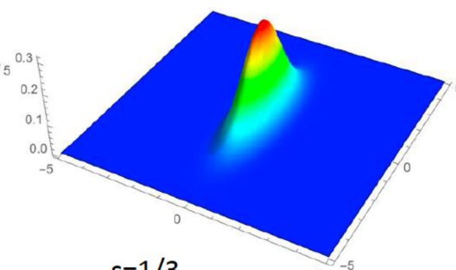
$$\hat{S}(\zeta) = e^{\frac{1}{2}\left(\zeta \hat{a}^{\dagger 2} - \zeta^* \hat{a}^2\right)}$$

$$|\zeta\rangle = \frac{1}{\sqrt{\cosh r}} \sum_{n=0}^{\infty} (-\tanh r)^n \frac{\sqrt{(2n)!}}{2^n n!} |2n\rangle$$

$$W_s(x,p) = \frac{1}{\pi} e^{-\frac{(x-\langle x\rangle)^2}{s} - \frac{(p-\langle p\rangle)^2}{1/s}}$$

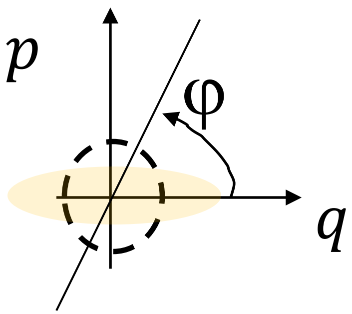


s=3

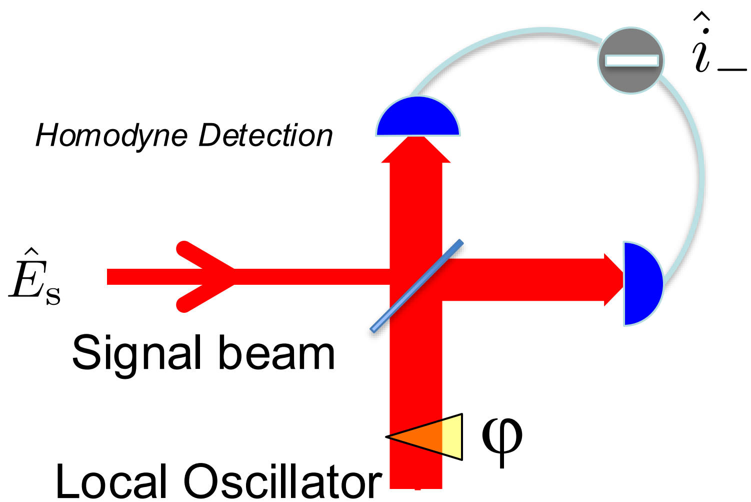


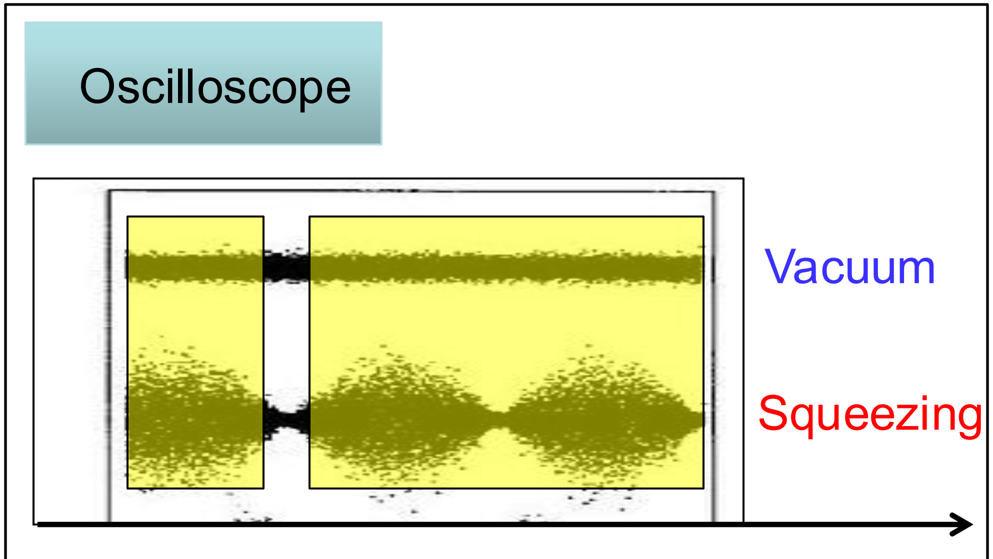
s=1/3

CV Quantum teleportation. The resource ?



squeezed state of light





Breitenbach *et al*., Nature (1997)

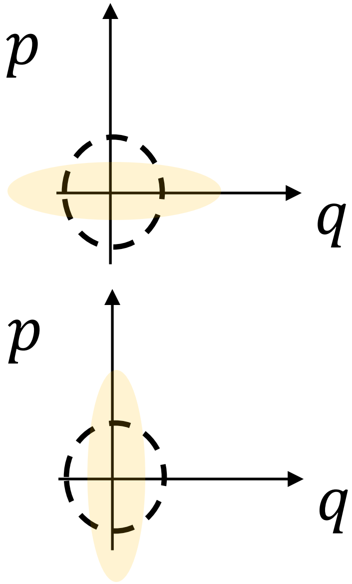
Projective measurement:

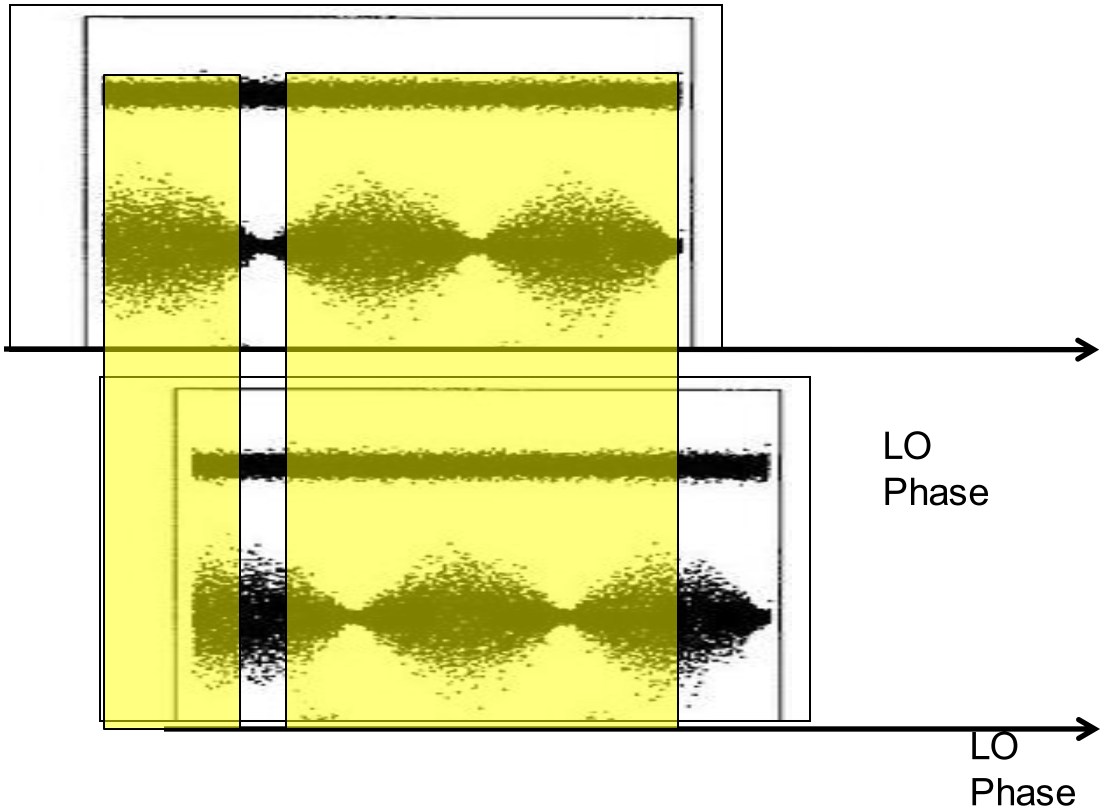
Measure directly the projection of Signal field on a quadrature of specific mode.

Squeezing = measured noise smaller than the vacuum noise

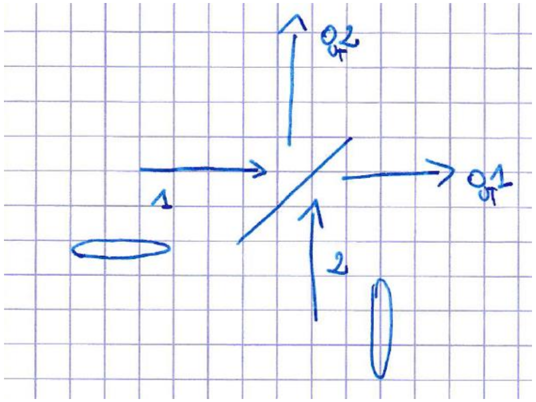
Squeezing usually expressed in dB

CV Quantum teleportation. The resource ?





CV Quantum teleportation. The resource ?



$$x_1 = x_0 e^{+r}$$

$$p_1 = p_0 e^{-r}$$

$$x_2 = x_0 e^{-r}$$

$$p_2 = p_0 e^{+r}$$

$$\begin{pmatrix} x_{out1} \\ x_{out2} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

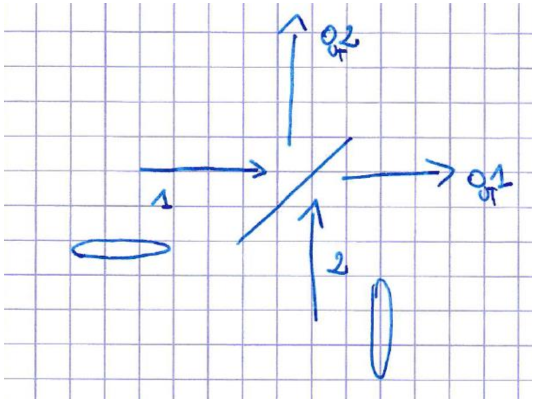
$$x_{out1} = \frac{1}{\sqrt{2}} x_0 \left(e^{+r} + e^{-r} \right)$$

$$x_{out2} = \frac{1}{\sqrt{2}} x_0 \left(-e^{+r} + e^{-r} \right)$$

$$p_{out1} = \frac{1}{\sqrt{2}} p_0 \left(e^{-r} + e^{+r} \right)$$

$$p_{out2} = \frac{1}{\sqrt{2}} p_0 \left(-e^{-r} + e^{+r} \right)$$

CV Quantum teleportation. The resource ?



$$x_1 = x_0 e^{+r} \qquad x_2 = x_0 e^{-r}$$

$$p_1 = p_0 e^{-r} \qquad p_2 = p_0 e^{+r}$$

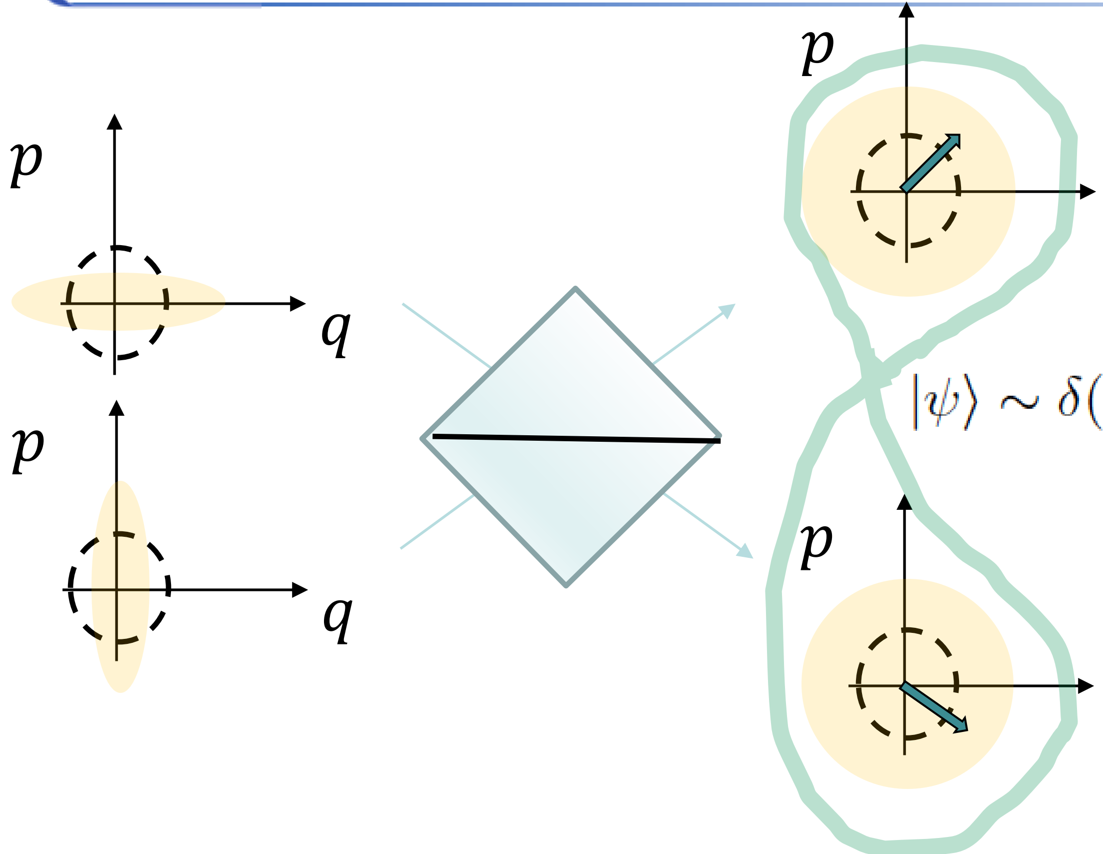
$$\begin{pmatrix} X_{out1} \\ X_{out2} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\Rightarrow \left\{ \begin{array}{l} X_{OUT1} - X_{OUT2} = \frac{2}{\sqrt{2}} x_0 \left(e^{r} \right) \\ \boxed{\begin{array}{l} p_{OUT1} - p_{OUT2} = \frac{2}{\sqrt{2}} p_0 e^{-r} \\ X_{OUT1} + X_{OUT2} = \frac{2}{\sqrt{2}} x_0 e^{-r} \end{array}} \\ p_{OUT1} + p_{OUT2} = \frac{2}{\sqrt{2}} p_0 e^{+r} \end{array} \right.$$

$$r \longrightarrow \infty$$

$$|\psi\rangle \sim \delta(q_s - q_i)\delta(p_s + p_i)$$

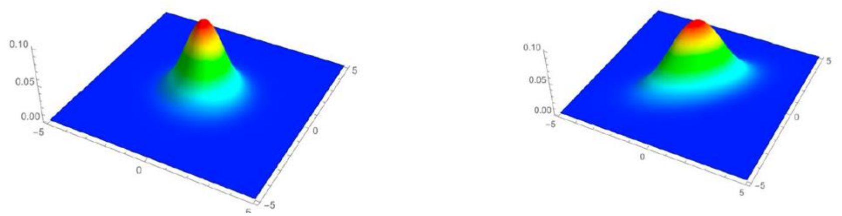
CV Quantum teleportation. The resource ?



$$|\psi\rangle \sim \delta(q_s - q_i)\delta(p_s + p_i)$$

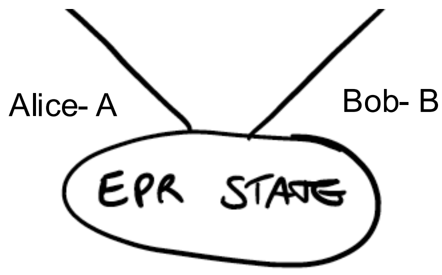
Two-mode squeezed vacuum =EPR state

$$|\psi_{EPR}\rangle = \frac{1}{\cosh r}\sum_{n=0}^{\infty} \tanh^n(r)\,|n,n\rangle$$



$$W_{EPR}(x_1,p_1,x_2,p_2) = \frac{1}{\pi^2} e^{-\frac{(x_1-x_2)^2}{2s} - \frac{(p_1-p_2)^2}{2/s} - \frac{(x_1+x_2)^2}{2/s} - \frac{(p_1+p_2)^2}{2s}}.$$

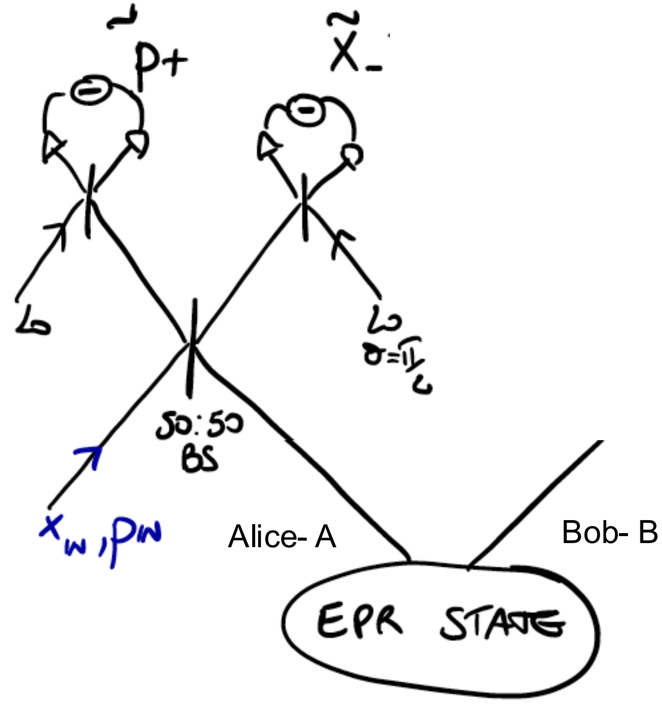
CV Quantum teleportation. The protocol



$$\Psi_{EPR} = \delta(x_A - x_B)\,\delta(p_A + p_B) \rightsquigarrow \begin{array}{l} x_A = x_B \\ p_A = -p_B \end{array}$$

Ideal case

CV Quantum teleportation. The protocol



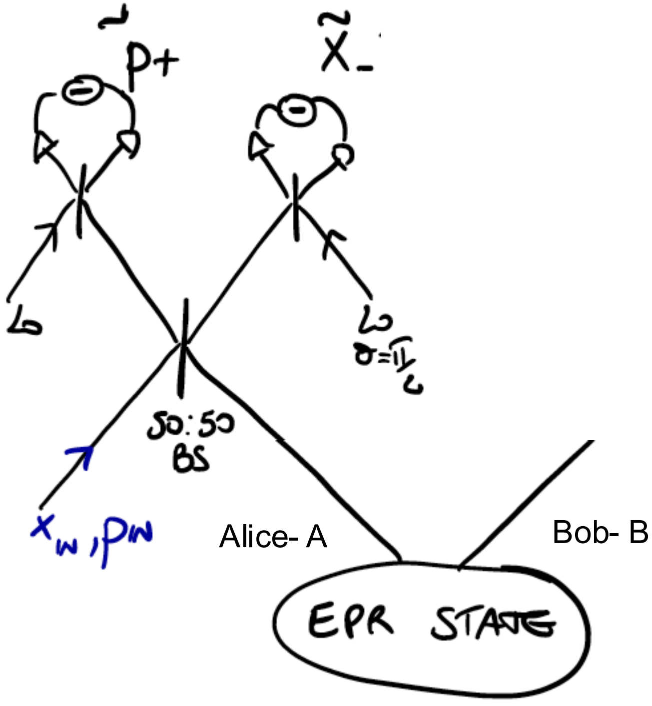
$$\Psi_{EPR} = \delta(x_A - x_B)\,\delta(p_A + p_B) \rightsquigarrow \begin{array}{l} x_A = x_B \\ p_A = -p_B \end{array}$$

Ideal case

Alice -> mixes the initial state with the EPR component, it measure via homodyne detection the two outputs

$$x_\pm = \frac{x_A \pm x_{in}}{\sqrt{2}} \qquad p^\pm = \frac{p_A + p_B}{\sqrt{2}}$$

CV Quantum teleportation. The protocol



$$\Psi_{EPR} = \delta(x_A - x_B)\,\delta(p_A + p_B) \rightsquigarrow \begin{array}{l} x_A = x_B \\ p_A = -p_B \end{array}$$

Ideal case

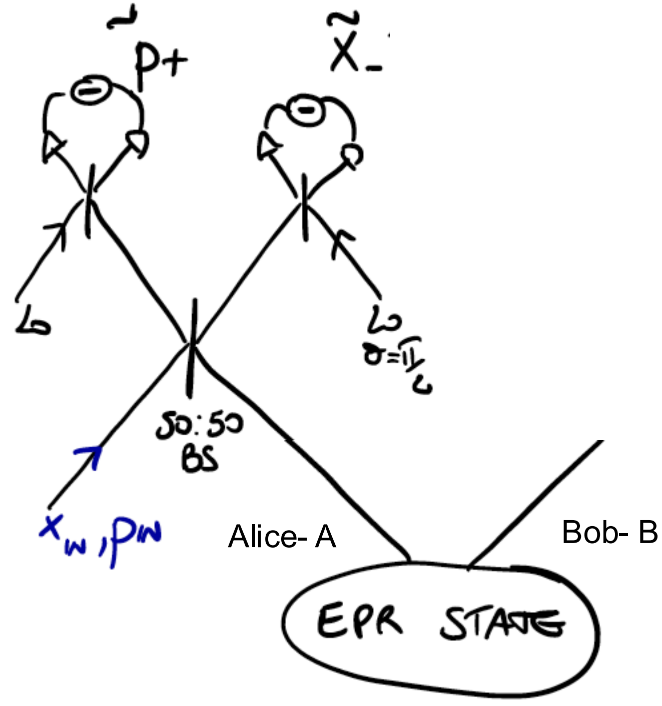
Alice -> mixes the initial state with the EPR component, it measure via homodyne detection the two outputs

She gets

$$\tilde{X}_- = \frac{x_A - x_{in}}{\sqrt{2}}$$

$$\tilde{P}_+ = \frac{p_A + p_{in}}{\sqrt{2}}$$

CV Quantum teleportation. The protocol



$$\Psi_{EPR} = \delta(x_A - x_B)\,\delta(p_A + p_B) \rightsquigarrow \begin{array}{l} x_A = x_B \\ p_A = -p_B \end{array}$$

Ideal case

Alice -> mixes the initial state with the EPR component, it measure via homodyne detection the two outputs

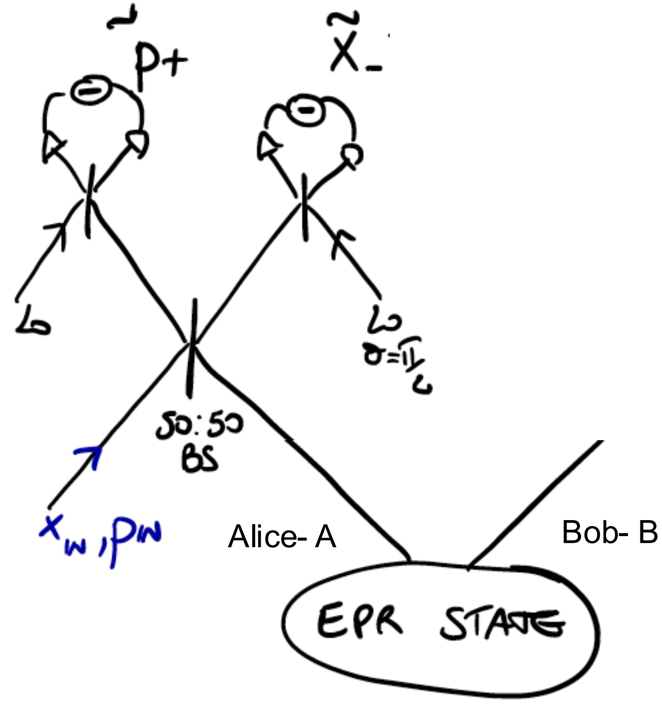
She gets

$$\tilde{X}_- = \frac{x_A - x_{in}}{\sqrt{2}} \rightsquigarrow x_A = \sqrt{2}\,\tilde{X}_- + x_{in}$$

projection

$$\tilde{p}_+ = \frac{p_A + p_{in}}{\sqrt{2}} \rightsquigarrow p_A = \sqrt{2}\,\tilde{p}_+ - p_{in}$$

CV Quantum teleportation. The protocol



$$\Psi_{EPR} = \delta(x_A - x_B)\,\delta(p_A + p_B) \rightsquigarrow \begin{matrix} x_A = x_B \\ p_A = -p_B \end{matrix}$$

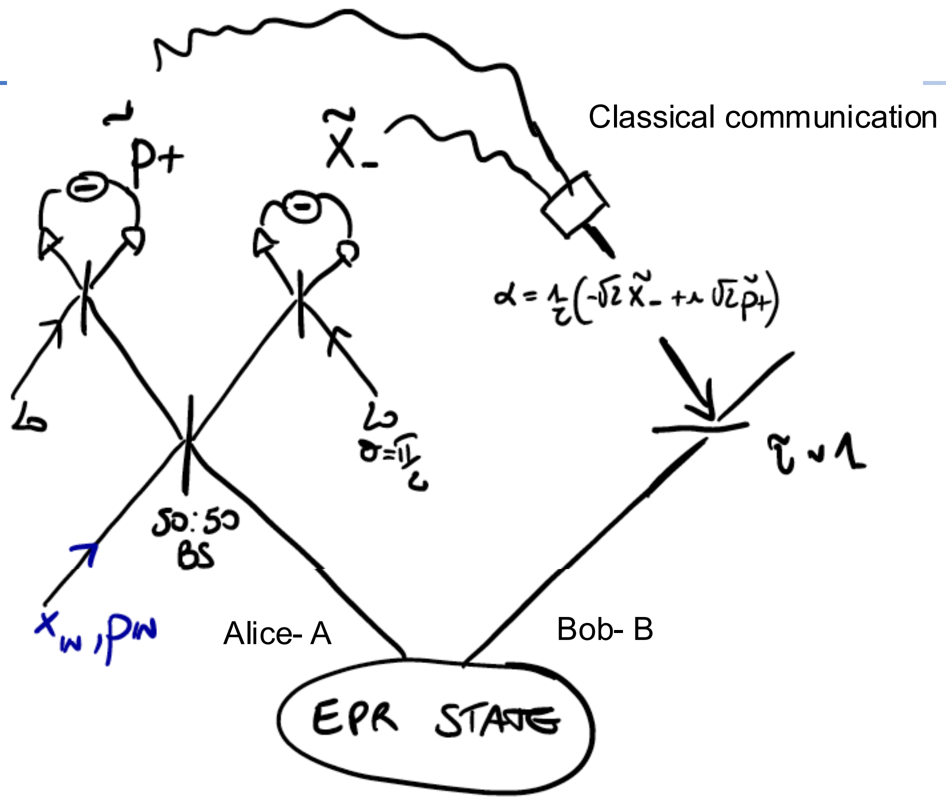
Ideal case

Alice -> mixes the initial state with the EPR component, it measure via homodyne detection the two outputs

She gets

$$\tilde{X}_- = \frac{x_A - x_{in}}{\sqrt{2}} \underset{\text{projection}}{\rightsquigarrow} x_A = \sqrt{2}\,\tilde{X}_- + x_{in} \underset{\text{projection}}{\rightsquigarrow} x_B = x_A = \sqrt{2}\,\tilde{X}_- + x_{in}$$

$$\tilde{p}_+ = \frac{p_A + p_{in}}{\sqrt{2}} \rightsquigarrow p_A = \sqrt{2}\,\tilde{p}_+ - p_{in} \rightsquigarrow p_B = -p_A = -\sqrt{2}\,\tilde{p}_+ + p_{in}$$



$$\Psi_{EPR} = \delta(x_A - x_B)\,\delta(p_A + p_B) \rightsquigarrow \begin{array}{l} x_A = x_B \\ p_A = -p_B \end{array}$$

Ideal case

Alice -> mixes the initial state with the EPR component, it measure via homodyne detection the two outputs

Alice –Bob classical communication

Bob-> Applies the corresponding displacement

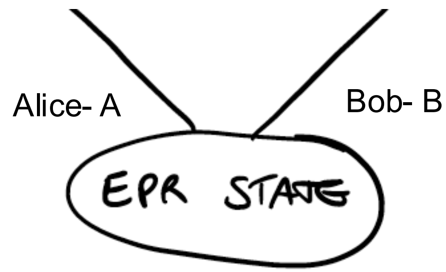
$$x_B = x_A = \sqrt{2}\,\tilde{x}_- + x_{in}$$

$$x_B' = x_B - \sqrt{2}\,\tilde{x}_- = x_{in} \,!$$

$$p_B' = p_B + \sqrt{2}\,\tilde{p}_+ = p_{in} \,!$$

$$p_B = -p_A = -\sqrt{2}\,\tilde{p}_+ + p_{in}$$

CV Quantum teleportation. The protocol



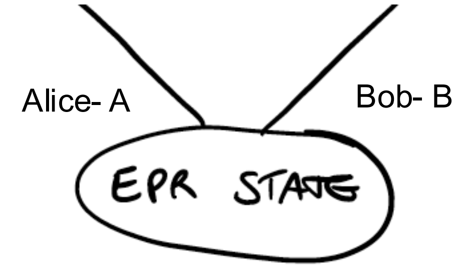
$$W_{EPR}(x_1,p_1,x_2,p_2) = \frac{1}{\pi^2} e^{-\frac{(x_1-x_2)^2}{2s} - \frac{(p_1-p_2)^2}{2/s} - \frac{(x_1+x_2)^2}{2/s} - \frac{(p_1+p_2)^2}{2s}}$$

NOT Ideal case

$$x_A - x_B = e^{-r}\left(x_A^0 - x_B^0\right)$$

$$p_A + p_B = e^{-r}\left(p_B^{(0)} + p_A^{(0)}\right)$$

CV Quantum teleportation. The protocol



$$W_{EPR}(x_1,p_1,x_2,p_2)=\frac{1}{\pi^2}\mathrm{e}^{-\frac{(x_1-x_2)^2}{2s}-\frac{(p_1-p_2)^2}{2/s}-\frac{(x_1+x_2)^2}{2/s}-\frac{(p_1+p_2)^2}{2s}}$$

NOT Ideal case

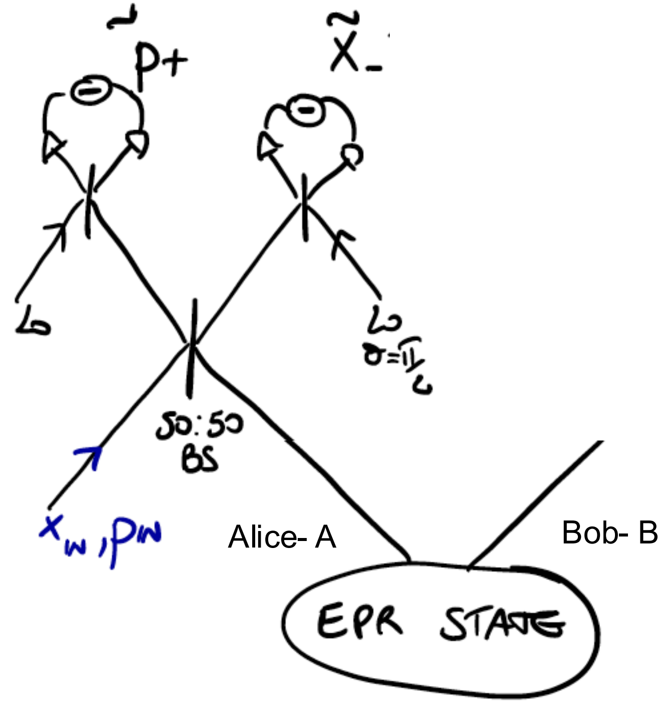
vacuum

$$x_A - x_B = e^{-\tau}\left(x_A^0 - x_B^0\right)$$

squeezing

$$p_A + p_B = e^{-\tau}\left(p_B^{(0)} + p_A^{(0)}\right)$$

CV Quantum teleportation. The protocol



$$W_{EPR}(x_1,p_1,x_2,p_2) = \frac{1}{\pi^2} e^{-\frac{(x_1-x_2)^2}{2s} - \frac{(p_1-p_2)^2}{2/s} - \frac{(x_1+x_2)^2}{2/s} - \frac{(p_1+p_2)^2}{2s}}$$

$$x_A - x_B = e^{-r}\left(x_A^{0} - x_B^{0}\right)$$

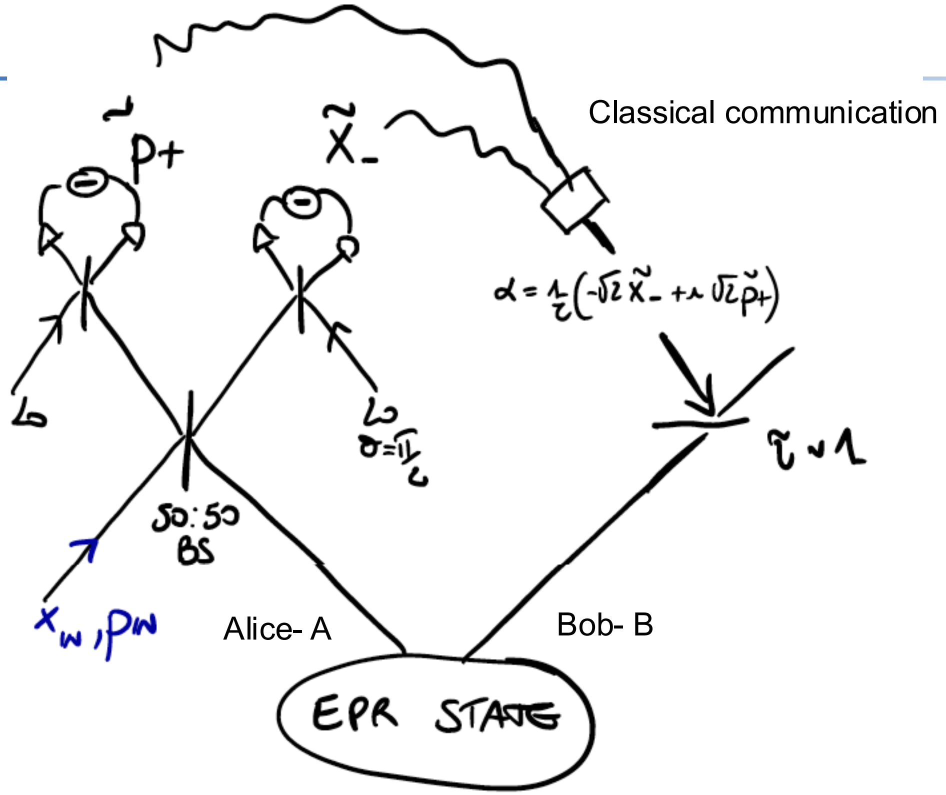
$$p_A + p_B = e^{-r}\left(p_B^{(0)} + p_A^{(0)}\right)$$

Not Ideal case

Alice -> mixes the initial state with the EPR component, it measure via homodyne detection the two outputs

$$x_B = x_A + \left(x_B^{(0)} - x_A^{(0)}\right)e^{-r} = \sqrt{2}\,\tilde{X}_- + x_{in} + \left(x_B^{(0)} - x_A^{(0)}\right)e^{-r}$$

$$p_B = -p_A + \left(p_B^{(0)} + p_A^{(0)}\right)e^{-r} = -\sqrt{2}\,\tilde{p}_+ + p_{in} + \left(p_B^{(0)} + p_A^{(0)}\right)e^{-r}$$



$$W_{EPR}(x_1,p_1,x_2,p_2) = \frac{1}{\pi^2} e^{-\frac{(x_1-x_2)^2}{2s} - \frac{(p_1-p_2)^2}{2/s} - \frac{(x_1+x_2)^2}{2/s} - \frac{(p_1+p_2)^2}{2s}}$$

$$x_A - x_B = e^{-\tau}(x_A^0 - x_B^0)$$

$$p_A + p_B = e^{-\tau}(p_B^{(0)} + p_A^{(0)})$$

Not Ideal case

Alice -> mixes the initial state with the EPR component, it measure via homodyne detection the two outputs

Alice –Bob classical communication

Bob-> Applies the corresponding displacement

$$x_B = x_{in} + (x_B^{(0)} - x_A^{(0)}) e^{-\tau}$$

$$p_B = p_{in} + (p_B^{(0)} + p_A^{(0)}) e^{-\tau}$$

Noise!